



The Islamia University Of Bahawalpur,
Department of Computer Science & IT
Bahawalnagar Campus

Course: Numerical Analysis **Program:** BSCS-V (Spring 2020)

Topic: Practice Questions (Polynomial Approximations)

lect-11

- practice Question -

16-3-17

No.3 Find the natural cubic spline which interpolate the data. Also find the $f'(6)$?

x	2	4	5	7
f(x)	1.34	1.84	1.12	0.02

Q: Find the best fit straight line to the following experimental data.

x	0.00	1.00	2.00	3.00	4.00
f(x)	1.00	3.85	6.50	9.35	12.05

Solution:

$h_1 = 2$; $h_2 = 1$; $h_3 = 2$

For a natural cubic spline we required $S''(x_1)$ and $S''(x_4)$ are zero.

The value of S'' at other data points are found from the system of equations

$$h_1 S''(x_1) + 2(h_1 + h_2) S''(x_2) + h_2 S''(x_3) = \frac{f_3 - f_1}{h_2} - \frac{f_2 - f_1}{h_1}$$

$$\Rightarrow 2(2+1) S''(x_2) + 1 S''(x_3) = 6 \left(\frac{f_3 - f_1}{h_2} - \frac{f_2 - f_1}{h_1} \right)$$

$$= 6 \left(\frac{1.12 - 1.34}{1} - \frac{1.84 - 1.34}{2} \right)$$

$$6 S''(x_2) + S''(x_3) = -5.82 \quad \text{--- (i)}$$

(65)

Now 2nd Eq.

$$h_2 S''(x_2) + 2(h_2 + h_3) S''(x_3) + h_3 S''(x_4) = \tau_3$$

$$1 S''(x_2) + 2(1 + 2) S''(x_3) = 6 \left(\frac{f_1 - f_3}{h_1} - \frac{f_3 - f_4}{h_2} \right)$$

$$= 6 \left(\frac{0.02 - 1.12}{2} - \frac{1.12 - 1.84}{1} \right)$$

$$S''(x_2) + 6 S''(x_3) = 1.02 \quad \text{--- (i)}$$

Multiplying by '6' above eq. we get

$$6 S''(x_2) + 36 S''(x_3) = 6.12 \quad \text{--- (ii)}$$

$$+ 6 S''(x_2) + S''(x_3) = -5.82$$

$$35 S''(x_3) = 11.94$$

$$S''(x_3) = \frac{11.94}{35}$$

$$S''(x_3) = 0.341143$$

By putting $S''(x_3)$ into eq (i)

$$6 S''(x_2) + 0.341143 = -5.82$$

$$= -5.82 - 0.341143$$

$$S''(x_2) = -6.161143 / 6$$

$$S''(x_2) = 1.02686$$

As we have to find $f(6)$

which lies in $5 < x < 7$

So first we find the spline for this interval

$$S'(x) = a_2(x - x_3)^3 + b_2(x - x_3)^2 + c_2(x - x_3) + d_2 \quad \text{--- (iii)}$$

Since

$$a_3 = \frac{s''(x_4) - s''(x_3)}{6h_3}$$

(66)

$$a_3 = \frac{-0.341143}{6 \times 2} = \frac{-0.341143}{12} = -0.02843$$

$$a_3 = -0.02843$$

$$b_3 = \frac{s''(x_3)}{2} = \frac{0.341143}{2}$$

$$b_3 = 0.170571$$

$$c_3 = \frac{f_4 - f_3}{h_3} - \left(\frac{s''(x_4) + 2s''(x_3)}{6} \right) h_3$$

$$= \frac{0.02 - 1.12}{2} - \left(\frac{2(0.341143)}{6} \right) 2$$

$$c_3 = -0.777429$$

$$d_4 = f_4 = 1.12$$

Substitute these values in eq (iv)

$$S(x) = 0.02843(x-5)^3 + 0.170571(x-5)^2$$

$$- 0.777429(x-5) + 1.12 \quad \Leftrightarrow (5 < x < 7)$$

For $f(6) = ?$

$x = 6$; use above spline $S(x)$

$$S(6) = -0.02843(6-5)^3 + 0.170571(6-5)^2$$

$$- 0.777429(6-5) + 1.12$$

$$S(6) = -0.02843 + 0.170571 - 0.777429 + 1.12$$

$$S(6) = -0.805859 + 1.290571 = 0.484712$$

(67)

(6)

As spline has a nice property

$$\rightarrow S(x) = f(x)$$

$$\cdot \text{So } S'(6) = f'(6) = 0.484712$$

\Rightarrow Find $P(3)$ for this data

$f(3)$ lies in interval $(2 < x < 4)$

Key-points.

$$\rightarrow S(x) = a_1(x-x_1)^3 + b_1(x-x_1)^2 + c_1(x-x_1) + d_1$$

\rightarrow Find a_1, b_1, c_1, d_1

\rightarrow We already ~~find~~ have values of

$$\underbrace{S'(x_1)}_{=0}, \underbrace{S'(x_2)}_{=1.02686}, \underbrace{S'(x_3)}_{=0.341143}, \underbrace{S'(x_4)}_{=0}$$

\uparrow Natural splines \uparrow

Q. Find The best fit straight line to the following experimental data

(68)

x	0.00	1.00	2.00	3.00	4.00
$f(x)$	1.00	3.85	6.50	9.35	12.05

sol

x_n	f_n	x_n^2	$x_n f_n$
10.00	32.75	30.00	93.10

$$\sum_{i=1}^n 1 = 5$$

$$c \sum_{i=1}^n 1 + m \sum_{i=1}^n x_n = \sum_{i=1}^n f_n \quad \text{--- (i)}$$

$$c \sum_{i=1}^n x_n + m \sum_{i=1}^n x_n^2 = \sum_{i=1}^n x_n f_n \quad \text{--- (ii)}$$

$$5c + 10m = 32.75 \quad \text{--- (A)}$$

$$10c + 30m = 93.10 \quad \text{--- (B)}$$

$$y = \underbrace{1.03}_{=c} + \underbrace{2.76x}_{=m}$$

(69)

7-3-17
FRIDAY

An experiment is carried out and the data obtained are as follows.

x_n	0.2	0.3	0.5	0.9
f_n	5.54	4.02	3.11	2.16

Find the least square best fit straight line $y = mx + c$

Sol.

x_n	f_n	x_n^2	$x_n f_n$
0.2	5.54	0.04	1.108
0.3	4.02	0.09	1.206
0.5	3.11	0.25	1.555
0.9	2.16	0.81	1.944
1.9	14.83	1.19	5.813

The pair of equations for m & c

$$c \sum_{i=1}^n 1 + m \sum_{i=1}^n x_i = \sum_{i=1}^n f_i$$

$$4c + 1.9 = 14.83 \quad \text{--- (i)}$$

And

$$c \sum_{i=1}^n x_i + m \sum_{i=1}^n x_i^2 = \sum_{i=1}^n x_i f_i$$

$$1.9c + 1.19m = 5.813 \quad \text{--- (ii)}$$

By solving the eq (i) & (ii), 70
we get

$$m = -4.28$$

$$\text{and } c = 5.74$$

So

$$y = mx + c$$

$$\boxed{y = -4.28x + 5.74}$$

Q No. 2

Use the polynomial approximation to estimate $f(22)$

x	x_1 10	x_2 15	x_3 20	x_4 25
$f(x)$	9.23	8.41	7.12	4.13

Sol. we apply the exact data method for $f(x)$

As first we find lagrange polynomial at $x = 22$

So we consider

$$L_1(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$

$$L_1(22) = \frac{(22-15)(22-20)(22-25)}{(10-15)(10-20)(10-25)} = 0.056$$

(71)

$$L_2(x) = \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$L_2(22) = \frac{(22-10)(22-20)(22-25)}{(25-10)(25-20)(15-25)} = -0.288$$

$$L_3(x) = \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)}$$

$$L_3(22) = \frac{(22-10)(22-15)(22-25)}{(20-10)(20-15)(20-25)} = 1.008$$

Similarly $L_4(22) = 0.224$

Now our required interpolated polynomial, at $x=22$ is

$$p(x) = f_1 L_1(x) + f_2 L_2(x) + f_3 L_3(x) + f_4 L_4(x)$$

i.e. $p(22) = f_1 L_1(22) + f_2 L_2(22) + f_3 L_3(22) + f_4 L_4(22)$

$$p(22) = 9.23 \times 0.056 + 8.41 \times -0.288 + 7.12 \times 1.008 + 4.13 \times 0.224$$

$$p(22) = 6.197$$

As $p(22) = f(22) = 6.20$

which is approximated value of $f(22)$

END-Lec

Best of Good luck